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No.	0	1	2	3	4
Series	$x'=1$;	x' ;	$Mx'-1$;	$[M^2-1]x'-M$;	$[M^3-2m]x'-M^2+1$;
			5	etc.	
			$[M^4-3M^2+1]x'-M^3+2M$;	etc.....	(6).

By restoring the value of x' , above=2, we have the series of values of the n th, middle side of the n th rational triangle in terms of the powers of magic M . The quantity in the powers of M multiplied by the constant factor x' , I call the "Continued Fraction Theorem" the laws of which are easily determined. These quantities are shown above in brackets in (6), and abbreviated to CFT.

Hence the middle side of the n th triangle= $\{[n\text{th.CFT}]2-(n-1)\text{th.CFT}\}2$(7). The laws of this theorem are: The 1st coefficient of $M=1$.

The 2nd coefficient of $M=\text{no. cycle}-2$.

The p th coefficient of $M=(\text{where } n-p=m)$

$$p\text{th coefficient} = \frac{m[m-1][m-2] \dots [m-p+2]}{1.2.3 \dots [p-1]} \dots (8).$$

If $n=10$, the 10th.CFT= $[M^9-8M^7+21M^5-20M^3+5M]$, and multiplied by $\frac{1}{2}M=x'=2$, from which take 9th.CFT= $M^8-7M^6+15M^4-10M^2+1$, the result multiplied by 2 will give the middle side, x , of the 10th rational triangle==524174.

Then the 10th triangle has for sides 524173, 524174, 524175.

AVERAGE AND PROBABILITY.

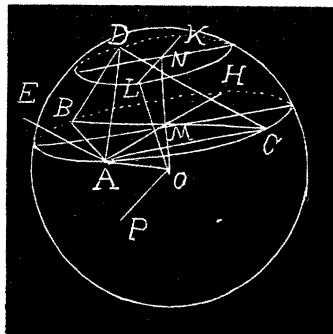
130. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Four points are taken at random on the surface of a given sphere; show that the average volume of a tetrahedron formed by the planes passing through the points taken three by three, is 1-35 of the volume of the given sphere.

II. Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A, B, C, D be the four random points; AH the diameter of the section through A, B, C ; LK the diameter of a section through D parallel to the section through A, B, C ; O the center of the sphere, M the center of AH , N the center of LK . Draw AE perpendicular to AH , and OP a line such that AB is parallel to the plane MOP . Let $AO=r$, $\angle AOM=\theta$, $\angle EAC=\phi$, $\angle EAB=\psi$, $\angle LON=\beta$, $\angle MOP=\lambda$, the angle POM makes with some fixed plane= ρ .

Then $AC=2r\sin\theta\sin\phi$, $AB=2r\sin\theta\sin\psi$; the volume, V , of the tetrahedron $DABO=\frac{2}{3}r^3(\cos\beta-\cos\theta)\sin^2\theta\sin\phi\sin\psi\sin(\phi-\psi)$. An element of surface at A is $2\pi r^2\sin\theta d\theta$; at D , $2\pi r^2\sin\beta d\beta$; at C , $4r^2\sin\theta\sin\phi d\phi d\lambda$; at B , $4r^2\sin\theta\sin(\phi-\psi)\sin\lambda\sin\psi d\psi d\rho$. The limits of θ are 0 and π ; of β , 0 and π ; of ϕ , 0 and π ; of ψ , 0 and ϕ and doubled; of λ , 0 and π ; of ρ , 0 and 2π . Since the whole number of ways the four points can be taken is $256\pi^4r^3$, we have



$$\begin{aligned}
\Delta &= \frac{2}{256\pi^4 r^8} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \int_0^\pi \int_0^{2\pi} V \cdot 2\pi r^2 \sin\theta d\theta \cdot 2\pi r^2 \sin\beta d\beta \cdot 4r^2 \sin\theta \sin\varphi d\varphi d\lambda \\
&\quad \times 4r^2 \sin\theta \sin(\varphi - \psi) \sin\lambda \sin\psi d\psi d\varphi \\
&= \frac{2r^8}{3\pi} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \int_0^\pi \sin^5\theta (\cos\beta - \cos\theta) \sin\beta \sin^2\varphi \sin^2\psi \sin^2(\varphi - \psi) \sin\lambda d\theta d\beta \\
&\quad \times d\varphi d\psi d\lambda \\
&= \frac{4r^8}{3\pi} \int_0^\pi \int_0^\theta \int_0^\pi \int_0^\phi \sin^5\theta \sin\beta (\cos\beta - \cos\theta) \sin^2\varphi \sin^2\psi \sin^2(\varphi - \psi) d\theta d\beta d\varphi d\psi \\
&= \frac{r^8}{6\pi} \int_0^\pi \int_0^\theta \int_0^\pi \sin^5\theta \sin\beta (\cos\beta - \cos\theta) (3\varphi \sin^2\varphi - 2\varphi \sin^4\varphi - 3\sin^5\varphi \cos\varphi) d\theta d\beta d\varphi \\
&= \frac{r^8}{16\pi} \int_0^\pi \int_0^\theta \sin^5\theta \sin\beta (\cos\beta - \cos\theta) d\theta d\beta \\
&= \frac{\pi r^8}{32} \int_0^\pi \sin^5\theta (2 - 2\cos\theta - \sin^2\theta) d\theta = (4\pi r^8/105) = \frac{1}{8} \text{ of volume of sphere.}
\end{aligned}$$

133. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

A circle of unknown radius is drawn with its center at the vertex of a given parabola, and has its greatest area when its circumference passes through the focus of the parabola. Required the average area common to the circle and parabola.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $y^2 = 4ax$, $x^2 + y^2 = r^2$ be the equation to the parabola and circle, respectively, $\Delta =$ average area, $x' = \sqrt{(r^2 + 4a^2)} - 2a$.

$$\begin{aligned}
\therefore \Delta &= 2 \int_0^a \left[2 \int_0^{x'} \sqrt{(ax)} dx + \int_{x'}^r \sqrt{(r^2 - x^2)} dx \right] dr / \int_0^a dr \\
&= \int_0^a \left[2 \int_0^{x'} \sqrt{(ax)} dx + \int_{x'}^r \sqrt{(r^2 - x^2)} dx \right] dr = \frac{1}{6a} \int_0^a \left[3\pi r^2 + 4\sqrt{(a)} [\sqrt{(r^2 + 4a^2)} \right. \\
&\quad \left. - 2a]^\frac{3}{2} - 6r^2 \sin^{-1} \left(\frac{\sqrt{(r^2 + 4a^2)} - 2a}{r} \right) \right] dr = \frac{1}{6} \pi a^2 - \frac{1}{3} a^2 \sin^{-1}(\sqrt{5} - 2) \\
&\quad + \frac{2}{3a} \int_0^a [\sqrt{(r^2 + 4a^2)} - 2a]^\frac{3}{2} dr + \frac{1}{3a} \int_0^a \frac{r^2 [\sqrt{(r^2 + 4a^2)} - 2a]^\frac{3}{2} dr}{\sqrt{(r^2 + 4a^2)}}.
\end{aligned}$$